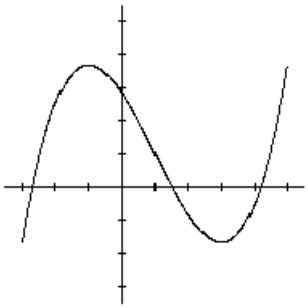
問題 7.1

(1) $y = \dfrac{1}{6}\left(x^3 - 3x^2 - 9x + 17\right) \quad (-3 \leqq x \leqq 5)$

$$\begin{aligned} y' &= \frac{1}{6}\left(3x^2 - 6x - 9\right) \\ &= \frac{1}{2}\left(x^2 - 2x - 3\right) \\ &= \frac{1}{2}(x+1)(x-3) \\ y'' &= \frac{1}{2}(2x-2) \\ &= x - 1 \end{aligned}$$

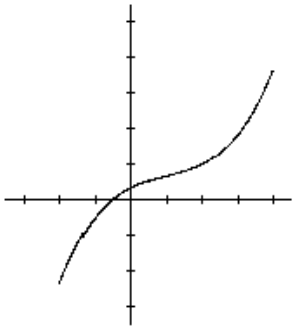


x	-3	$\cdots$	-1	$\cdots$	1	$\cdots$	3	$\cdots$	5
y'	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$
y''	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$
y	$-\frac{5}{3}$	◜	$\frac{11}{3}$	◝	1	◟	$-\frac{5}{3}$	◞	$\frac{11}{3}$

極大点 ： $\left(-1, \dfrac{11}{3}\right)$　　極小点 ： $\left(3, -\dfrac{5}{3}\right)$　　変曲点 ： $(1, 1)$

(2) $y = \dfrac{1}{12}x^3 - \dfrac{1}{4}x^2 + \dfrac{1}{2}x + \dfrac{1}{3} \quad (-2 \leqq x \leqq 4)$

$$\begin{aligned} y' &= \frac{1}{4}x^2 - \frac{1}{2}x + \frac{1}{2} \\ &= \frac{1}{4}(x^2 - 2x + 2) \\ &= \frac{1}{4}\left((x-1)^2 + 1\right) \\ &> 0 \quad (\text{常に}) \\ y'' &= \frac{1}{2}x - \frac{1}{2} \\ &= \frac{1}{2}(x-1) \end{aligned}$$

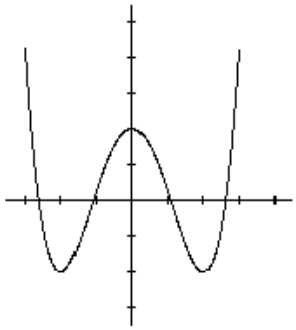


x	-2	$\cdots$	1	$\cdots$	4
y'	$+$	$+$	$+$	$+$	$+$
y''	$-$	$-$	0	$+$	$+$
y	$-\frac{7}{3}$	◜	$\frac{2}{3}$	◞	$\frac{11}{3}$

極大点 ： なし　　極小点 ： なし　　変曲点 ： $\left(1, \dfrac{2}{3}\right)$

(3) $y = \dfrac{1}{4}x^4 - 2x^2 + 2 \quad (-3 \leqq x \leqq 3)$

$$\begin{aligned} y' &= x^3 - 4x \\ &= x(x-2)(x+2) \\ y'' &= 3x^2 - 4 \\ &= (\sqrt{3}x - 2)(\sqrt{3}x + 2) \end{aligned}$$

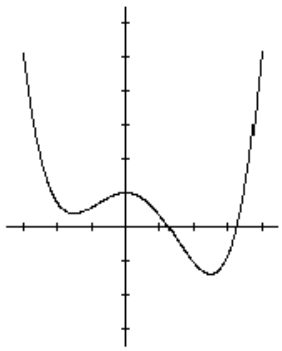


x	-3	$\cdots$	-2	$\cdots$	$-\dfrac{2}{\sqrt{3}}$	$\cdots$	0	$\cdots$	$\dfrac{2}{\sqrt{3}}$	$\cdots$	2	$\cdots$	3
y'	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$
y''	$+$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$
y	$\dfrac{17}{4}$	◟	-2	◞	$-\dfrac{2}{9}$	◜	2	◝	$-\dfrac{2}{9}$	◟	-2	◞	$\dfrac{17}{4}$

極大点　：　$(0, 2)$　　極小点　：　$(-2, -2), (2, -2)$　　変曲点　：　$\left(-\dfrac{2}{\sqrt{3}}, -\dfrac{2}{9}\right), \left(\dfrac{2}{\sqrt{3}}, -\dfrac{2}{9}\right)$

(4) $y = \dfrac{1}{12}x^4 - \dfrac{1}{9}x^3 - \dfrac{5}{8}x^2 + 1 \quad (-3 \leqq x \leqq 4)$

$$\begin{aligned} y' &= \frac{1}{3}x^3 - \frac{1}{3}x^2 - \frac{5}{4}x \\ &= \frac{1}{12}x(4x^2 - 4x - 15) \\ &= \frac{1}{12}x(2x+3)(2x-5) \\ y'' &= x^2 - \frac{2}{3}x - \frac{5}{4} \\ &= \frac{1}{12}(12x^2 - 8x - 15) \\ &= \frac{1}{12}(6x+5)(2x-3) \end{aligned}$$



x	-3	$\cdots$	$-\dfrac{3}{2}$	$\cdots$	$-\dfrac{5}{6}$	$\cdots$	0	$\cdots$	$\dfrac{2}{3}$	$\cdots$	$\dfrac{5}{2}$	$\cdots$	4
y'	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$
y''	$+$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$
y		◟	極小	◞	変曲	◜	極大	◝	変曲	◟	極小	◞	

極大点　：　$(0,\quad)$

極小点　：　$\left(-\dfrac{3}{2},\quad\right), \left(\dfrac{5}{2},\quad\right)$　　変曲点　：　$\left(-\dfrac{5}{6},\quad\right), \left(\dfrac{2}{3},\quad\right)$