

## 1 $n$ 回の試行で偶数回起こる確率

**問題 1.1** 1 回の試行で事象  $A$  が起こる確率を  $p$  とする.  $n$  回の試行で  $A$  が偶数回起こる確率  $E_n$  を求めなさい.

**解答 1.1**

$$E_0 = 1$$

$$E_1 = 1 - p$$

$$E_{k+1} = (1 - p) E_k + p (1 - E_k) = (1 - 2p)E_k + p$$

$$E_{k+1} - \frac{1}{2} = (1 - 2p) \left( E_k - \frac{1}{2} \right)$$

$$\therefore E_n = \frac{1}{2} + (1 - 2p)^n \left( E_0 - \frac{1}{2} \right) = \frac{1}{2} (1 + (1 - 2p)^n)$$

**解答 1.2**  $q = 1 - p$ ,  $n$  回の試行で  $A$  が奇数回起こる確率を  $O_n$  とする.

$$E_n = p^0 q^n + {}_n C_2 p^2 q^{n-2} + \dots + {}_n C_{2k} p^{2k} q^{n-2k} + \dots$$

$$O_n = {}_n C_1 p^1 q^{n-1} + {}_n C_3 p^3 q^{n-3} + \dots + {}_n C_{2k+1} p^{2k+1} q^{n-2k-1} + \dots$$

$$E_n + O_n = \sum_{i=0}^n {}_n C_i p^i q^{n-i} = (p + q)^n = 1$$

$$E_n - O_n = \sum_{i=0}^n {}_n C_i (-p)^i q^{n-i} = (-p + q)^n$$

$$\therefore E_n = \frac{1}{2} (1 + (-p + q)^n)$$