公式（三角比）

内角の和

$$A+B+C=180^\circ$$

余角公式

$$\sin(90^\circ-A) = \cos A \qquad \cos(90^\circ-A) = \sin A \qquad \tan(90^\circ-A) = \frac{1}{\tan A}$$

補角公式

$$\sin(180^\circ-A) = +\sin A \qquad \cos(180^\circ-A) = -\cos A \qquad \tan(180^\circ-A) = -\tan A$$

相互関係

$$\tan A=\frac{\sin A}{\cos A} \qquad \sin A=\tan A\cos A \qquad \cos A=\frac{\sin A}{\tan A}$$

$$\cos^2 A+\sin^2 A = 1 \qquad 1+\tan^2 A = \frac{1}{\cos^2 A} \qquad \frac{1}{\tan^2 A}+1 = \frac{1}{\sin^2 A}$$

正弦定理 1

$$a:b:c=\sin A:\sin B:\sin C$$

正弦定理 2

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R \qquad (R\text{ は外接円の半径})$$

余弦定理 1

$$a=b\cos C+c\cos B \qquad b=c\cos A+a\cos C \qquad c=a\cos B+b\cos A$$

余弦定理 2

$$a^2=b^2+c^2-2bc\cos A \qquad b^2=c^2+a^2-2ca\cos B \qquad c^2=a^2+b^2-2bc\cos C$$

$$\cos A=\frac{b^2+c^2-a^2}{2bc} \qquad \cos B=\frac{c^2+a^2-b^2}{2ca} \qquad \cos C=\frac{a^2+b^2-c^2}{2ab}$$

三角形の形状

$$A<90^\circ \iff a^2<b^2+c^2 \qquad A=90^\circ \iff a^2=b^2+c^2 \qquad A>90^\circ \iff a^2>b^2+c^2$$

$$\begin{aligned}
\text{鋭角三角形} &\iff (a^2+b^2-c^2)(b^2+c^2-a^2)(c^2+a^2-b^2)>0\\
\text{直角三角形} &\iff (a^2+b^2-c^2)(b^2+c^2-a^2)(c^2+a^2-b^2)=0\\
\text{鈍角三角形} &\iff (a^2+b^2-c^2)(b^2+c^2-a^2)(c^2+a^2-b^2)<0\\
&\quad\ \text{かつ}\ \ (a+b-c)(b+c-a)(c+a-b)>0
\end{aligned}$$

面積の公式

$$S=\frac{1}{2}bc\sin A \qquad S=\frac{1}{2}ca\sin B \qquad S=\frac{1}{2}ab\sin C$$

$$S=\frac{abc}{4R}$$

$$S=\frac{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}{4}$$

$$S=\sqrt{s(s-a)(s-b)(s-c)} \qquad \text{ただし} \quad s=\frac{a+b+c}{2}$$

基本的な角の三角比

	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	×	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

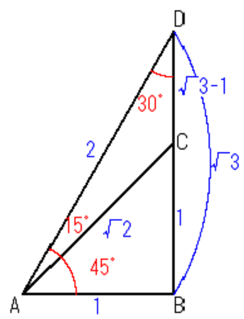
15°,75°の三角比

三角形 ACD においてにおいて

$$\sin 15^\circ : \sin 30^\circ = \sqrt{3}-1 : \sqrt{2}$$

$$\sin 15^\circ = \frac{\sqrt{3}-1}{\sqrt{2}}\sin 30^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos 15^\circ = \frac{2^2+\sqrt{2}^2-(\sqrt{3}-1)^2}{2\cdot 2\cdot\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

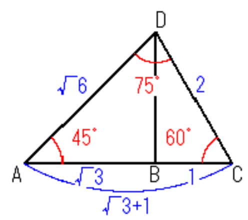


三角形 DAC において

$$\sin 75^\circ : \sin 45^\circ = \sqrt{3}+1 : 2$$

$$\sin 75^\circ = \frac{\sqrt{3}+1}{2}\sin 45^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

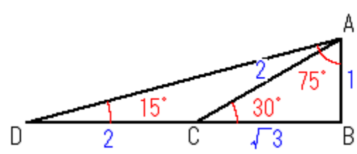
$$\cos 75^\circ = \frac{2^2+\sqrt{6}^2-(\sqrt{3}+1)^2}{2\cdot 2\cdot\sqrt{6}} = \frac{3-\sqrt{3}}{2\sqrt{6}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$



三角形 ADB において

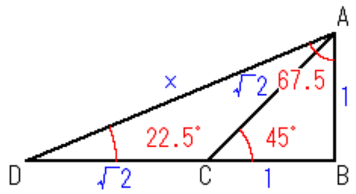
$$\tan 15^\circ = \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$$

$$\tan 75^\circ = 2+\sqrt{3}$$



22.5°,67.5° の三角比

三角形 ADB において



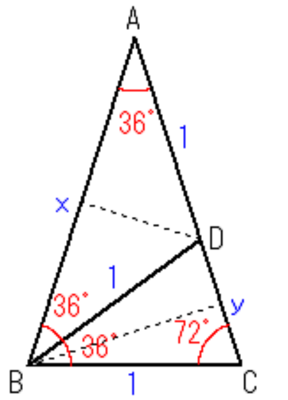
$$\tan 22.5^\circ = \frac{1}{\sqrt{2}+1} = \sqrt{2}-1$$
$$\tan 67.5^\circ = \sqrt{2}+1$$
$$x = \sqrt{(\sqrt{2}+1)^2+1^2} = \sqrt{2(2+\sqrt{2})}$$
$$\sin 22.5^\circ = \cos 67.5^\circ = \frac{1}{x} = \frac{1}{\sqrt{2(2+\sqrt{2})}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

二等辺三角形 CDA において

$$\sin 67.5^\circ = \cos 22.5^\circ = \frac{x}{2\sqrt{2}} = \frac{\sqrt{2(2+\sqrt{2})}}{2\sqrt{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

18°,36°,54°,72° の三角比

二等辺三角形 ABC と二等辺三角形 BCD は相似だから



$$x : 1 = 1 : y = 1 : x-1$$
$$x(x-1) = 1 \cdot 1$$
$$x^2 - x - 1 = 0$$
$$\therefore \quad x = \frac{\sqrt{5}+1}{2}, \quad y = \frac{\sqrt{5}-1}{2}$$

二等辺三角形 BCD において

$$\sin 18^\circ = \cos 72^\circ = \frac{y}{2} = \frac{\sqrt{5}-1}{4}$$
$$\sin 72^\circ = \cos 18^\circ = \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2} = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

二等辺三角形 DAB において

$$\sin 54^\circ = \cos 36^\circ = \frac{x}{2} = \frac{\sqrt{5}+1}{4}$$
$$\sin 36^\circ = \cos 54^\circ = \sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)^2} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$