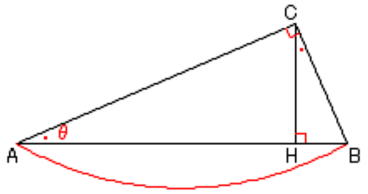
1　三角関数の公式の図解

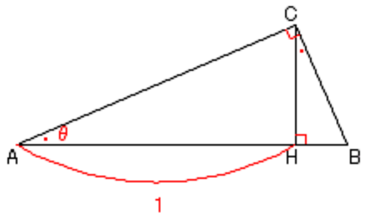
三角関数の公式の多くは基本の公式から式変形によって得られます．こうして計算で得られた公式を憶えようとしてもすぐ忘れてしまいがちです．図を用いて視覚的に確認することによって忘れにくくなります．

注 1.1　図解は一般角についての証明としては不十分です．

1.1　相互関係

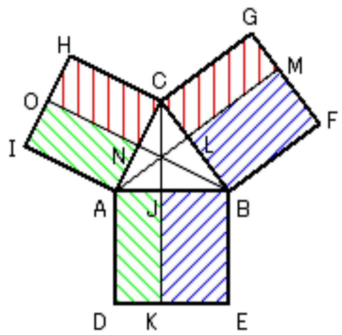


$$\mathrm{AH} = \mathrm{AC}\cos\theta = \cos^2\theta$$
$$\mathrm{BH} = \mathrm{BC}\sin\theta = \sin^2\theta$$
$$\therefore\ \cos^2\theta + \sin^2\theta = \mathrm{AH} + \mathrm{BH} = \mathrm{AB} = 1$$



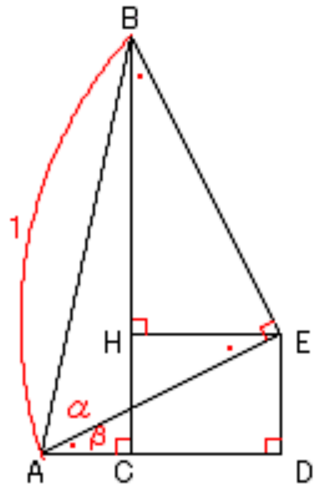
$$\mathrm{AB} = \frac{\mathrm{AC}}{\cos\theta} = \frac{1}{\cos^2\theta}$$
$$\mathrm{BH} = \mathrm{CH}\tan\theta = \tan^2\theta$$
$$\therefore\ \frac{1}{\cos^2\theta} = 1 + \tan^2\theta$$

1.2　余弦定理



$$\begin{aligned} a^2 &= \square\mathrm{BCGF} \\ &= \square\mathrm{CGML} + \square\mathrm{BFML} \\ &= \square\mathrm{CHON} + \square\mathrm{BEKJ} \\ &= \square\mathrm{CAIH} - \square\mathrm{AION} + \square\mathrm{ABED} - \square\mathrm{ADKJ} \\ &= b^2 - bc\cos A + c^2 - cb\cos A \\ &= b^2 + c^2 - 2bc\cos A \end{aligned}$$

1.3　加法定理



$\mathrm{BC} = \sin(\alpha + \beta)$

$\mathrm{BH} = \mathrm{BE}\cos\beta = \sin\alpha\cos\beta$

$\mathrm{HC} = \mathrm{ED} = \mathrm{AE}\sin\beta = \cos\alpha\sin\beta$

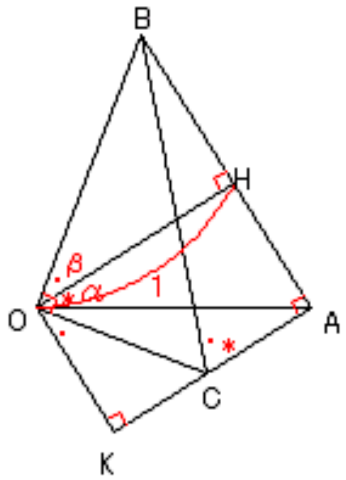
$\therefore\ \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\mathrm{AC} = \cos(\alpha + \beta)$

$\mathrm{AD} = \mathrm{AE}\cos\beta = \cos\alpha\cos\beta$

$\mathrm{CD} = \mathrm{HE} = \mathrm{BE}\sin\beta = \sin\alpha\sin\beta$

$\therefore\ \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$



OH から始めて，三角形 OAB，長方形 OHAK をかいて，$\angle\mathrm{BOC} =$ 直角 のように C をとる．

$\angle\mathrm{ACB} = \angle\mathrm{AOB} = \alpha + \beta$

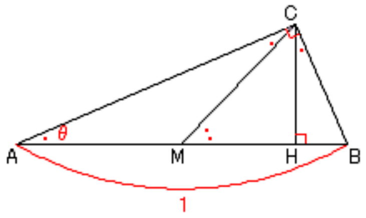
$\dfrac{\mathrm{AB}}{\mathrm{AC}} = \tan(\alpha + \beta)$

$\mathrm{AB} = \mathrm{AH} + \mathrm{HB} = \tan\alpha + \tan\beta$

$\mathrm{AC} = \mathrm{AK} - \mathrm{KC} = 1 - \tan\alpha\ \tan\beta$

$$\therefore\ \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\ \tan\beta}$$

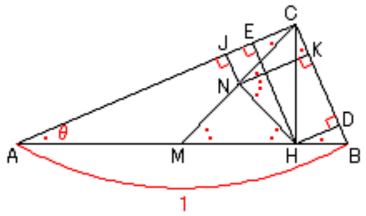
1.4　半角公式（倍角公式の逆）



$\angle$ACB = 直角, AM = BM, CH $\perp$ AB

$$\mathrm{AM} = \mathrm{BM} = \mathrm{CM} = \frac{1}{2}$$
$$\angle \mathrm{BMC} = 2\theta$$
$$\mathrm{MH} = \frac{1}{2}\cos 2\theta$$
$$\mathrm{BH} = \mathrm{BC}\sin\theta = \sin^2\theta$$
$$\therefore\ \sin^2\theta = \frac{1-\cos 2\theta}{2}$$
$$\mathrm{AH} = \mathrm{AC}\cos\theta = \cos^2\theta$$
$$\therefore\ \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

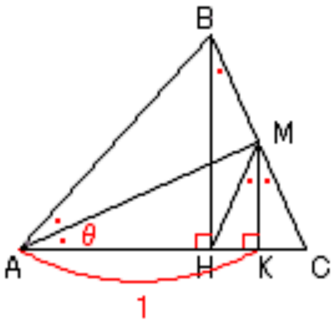
1.5　3 倍角公式の逆



$\angle$ACB = 直角, AM = BM, CH $\perp$ AB

$$\mathrm{CN} = \mathrm{MN} = \mathrm{HN} = \frac{1}{2}\mathrm{CM} = \frac{1}{4}$$
$$\angle \mathrm{HNC} = 2\angle \mathrm{HMC} = 4\theta$$
$$\angle \mathrm{HNK} = 3\theta$$
$$\mathrm{BD} = \mathrm{BH}\sin\theta = \sin^3\theta$$
$$\mathrm{CK} = \mathrm{CN}\sin\theta = \frac{1}{4}\sin\theta$$
$$\mathrm{DK} = \mathrm{HN}\sin 3\theta = \frac{1}{4}\sin 3\theta$$
$$\therefore\ \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$
$$\mathrm{AE} = \mathrm{AH}\cos\theta = \cos^3\theta$$
$$\mathrm{CJ} = \mathrm{CN}\cos\theta = \frac{1}{4}\cos\theta$$
$$\mathrm{EJ} = \mathrm{HN}\cos 3\theta = \frac{1}{4}\cos 3\theta$$
$$\therefore\ \cos^3\theta = \frac{3\cos\theta + \cos 3\theta}{4}$$

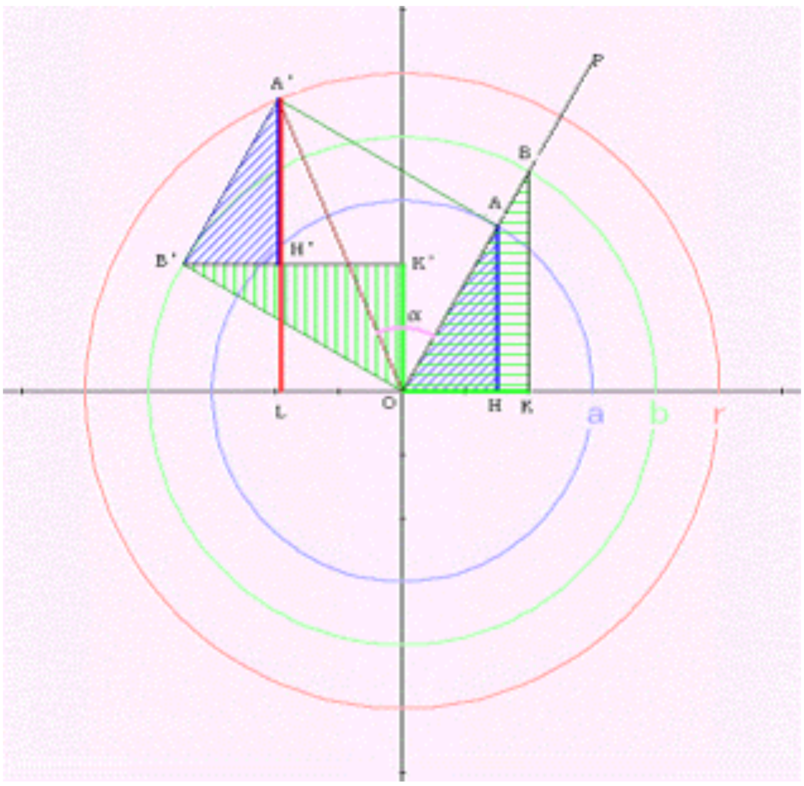
1.6　tan で表す倍角公式



AB = AC, AM ⊥ BC

$$
\begin{aligned}
&\mathrm{MH} = \mathrm{MB} = \mathrm{MC}\\
&\mathrm{HK} = \mathrm{CK} = \mathrm{MK}\tan\theta = \tan^2\theta\\
&\mathrm{AH} = \mathrm{AK} - \mathrm{HK} = 1 - \tan^2\theta\\
&\mathrm{BH} = 2\mathrm{MK} = 2\tan\theta\\
&\mathrm{AB} = \mathrm{AC} = \mathrm{AK} + \mathrm{CK} = 1 + \tan^2\theta\\
&\therefore\ \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\\
&\quad\ \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}\\
&\quad\ \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}
\end{aligned}
$$

1.7　合成（加法定理の逆）



$a\sin\theta + b\cos\theta = \mathrm{AH} + \mathrm{OK} = \mathrm{A'H'} + \mathrm{OK'} = \mathrm{A'L} = r\sin(\theta+\alpha)$

ただし，三角形OAA′ より

$$r = \sqrt{a^2+b^2},\quad \cos\alpha = \frac{a}{r},\quad \sin\alpha = \frac{b}{r}$$