

3 変数対称式

$$P = x + y + z$$

$$Q = xy + yz + zx$$

$$R = xyz$$

で表す。

$$\begin{aligned}x^2 + y^2 + z^2 &= (x + y + z)^2 - 2(xy + yz + zx) \\ &= P^2 - 2Q\end{aligned}$$

$$\begin{aligned}(x - y)^2 + (y - z)^2 + (z - x)^2 &= 2(x^2 + y^2 + z^2 - xy - yz - zx) \\ &= 2(P^2 - 3Q)\end{aligned}$$

$$\begin{aligned}x^2y + xy^2 + y^2z + yz^2 + z^2x + zx^2 &= (x + y + z)(xy + yz + zx) - 3xyz \\ &= PQ - 3R\end{aligned}$$

$$\begin{aligned}x^3 + y^3 + z^3 &= (x + y + z)(x^2 + y^2 + z^2) \\ &\quad - (x^2y + xy^2 + y^2z + yz^2 + z^2x + zx^2) \\ &= P^3 - 3PQ + 3R\end{aligned}$$

$$\begin{aligned}x^2y^2 + y^2z^2 + z^2x^2 &= (xy + yz + zx)^2 - 2(x + y + z)xyz \\ &= Q^2 - 2PR\end{aligned}$$

$$\begin{aligned}x^3y + xy^3 + y^3z + yz^3 + z^3x + zx^3 &= (x^2 + y^2 + z^2)(xy + yz + zx) - (x + y + z)xyz \\ &= P^2Q - 2Q^2 + PR\end{aligned}$$

$$\begin{aligned}x^4 + y^4 + z^4 &= (x^2 + y^2 + z^2)^2 - 2(x^2y^2 + y^2z^2 + z^2x^2) \\ &= (P^2 - 2Q)^2 - 2(Q^2 - 2PR) \\ &= P^4 - 4P^2Q + 2Q^2 + 4PR\end{aligned}$$

$$\begin{aligned}(x - y)^4 + (y - z)^4 + (z - x)^4 &= 2(x^4 + y^4 + z^4) - 4(x^3y + xy^3 + y^3z + yz^3 + z^3x + zx^3) \\ &\quad + 6(x^2y^2 + y^2z^2 + z^2x^2) \\ &= 2P^4 - 12P^2Q + 18Q^2 - 8PR\end{aligned}$$